

*v* Spreads Due to Systematic Field Errors  
Including  $b_3$ ,  $b_4$ , and  $a_1$

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## $b_4$ effects

Systematic  $b_4$  due to iron saturation at high fields

$$b_4' = 6 \times 10^{-4}$$

$$b_4 = 1.54 \times 10^{-5} / \text{cm}^{-4}$$

Reference: AD/RHIC-28, Dell, Hahn, Perzen, Ruggiero

## $\nu$ -Shifts

For each magnet type

$$\Delta\nu_x = \frac{1}{\pi} \frac{N}{P} L b_4 \left\{ \beta_x (x_{pf})^3 + .75 \beta_x^2 x_{pf} \epsilon_x - 1.5 \beta_x \beta_y (x_{pf}) \epsilon_y \right\}$$

$$\Delta\nu_y = \frac{1}{\pi} \frac{N}{P} L b_4 \left\{ -\beta_y (x_{pf})^3 - 1.5 \beta_x \beta_y \epsilon_x x_{pf} + .75 \beta_y^2 x_{pf} \epsilon_y \right\}$$

$$\delta = \sigma P/P$$

$\Delta\nu$  depends on  $\delta, \epsilon_x, \epsilon_y$  — may take as many as 6 power supplies to control the 6 different terms in  $\Delta\nu_x, \Delta\nu_y$ . With 2 power supplies,  $\Delta\nu$  can be reduced by about a factor of 2.

## b<sub>4</sub> effects and correction

$$b'_4 = 6, b_4 = 1.54 \times 10^{-5} \text{ cm}^{-4}, \gamma = 100$$

|  | Old RF | New RF                |                       |
|--|--------|-----------------------|-----------------------|
|  |        | $\epsilon_{x_0} = 10$ | $\epsilon_{x_0} = 60$ |
| $\Gamma_x$ (mm)  |        | 1.7                   | 2.4                   |
| $\epsilon_{x,N}$   | 30     | 35                    | 69                    |
| $\epsilon_x = 6\Gamma_x^2/B_x$                             | .3     | .35                   | .69                   |
| $\epsilon_{b_f} = 6\epsilon_x$                             | 1.8    | 2.1                   | 4.1                   |
| $\delta p/10^{-3}$   | 1.04   | 1.58                  | .993                  |
| $\frac{\delta p}{p}/10^{-3} = \frac{2.5\delta p}{10^{-3}}$ | 2.6    | 3.95                  | 2.5                   |
| $\Delta V/10^{-3}$   | 11     | 21                    | 22                    |
| To be corrected  |        |                       |                       |
| $\Delta V/10^{-3}$   | 5      | 10                    | 11                    |
| Corrected  |        |                       |                       |
| $b_{4L} (m^{-3})/10$                                       | 12.5   | 12.7                  | 12.4                  |
| Correction needed  |        |                       |                       |

$$\Delta V_{\text{spread}} = 2 \times \Delta V \quad \text{because } \Delta V \approx \delta p/p$$

$$\Delta V_{\text{spread}} = 4.0 \times 10^{-3} \quad \text{uncorrected}$$

$$= 2.0 \times 10^{-3} \quad \text{corrected}$$

$$\text{For } \epsilon_{2.50} = .35, \quad \delta p/p = 2\delta p = 3.2 \times 10^{-3} \quad \left. \right\} \epsilon_{x_0} = 10$$

$$\Delta V_{\text{spread}} = 8.2 \times 10^{-3} \quad \text{uncorrected}$$

$$\Delta V_{\text{spread}} = 4 \times 10^{-3} \quad \text{corrected}$$

About same result for  $\epsilon_{x_0} = 60$

(3)

## $b_3$ effects

There is no known source of systematic  $b_3$ . Fermilab experience indicates that construction procedures introduce an average  $b_3$  which is not zero; and

$$b_{3,av} \approx \frac{1}{3} b_{3,rms},$$

where  $b_{3,rms}$  is the rms random  $b_3$ .

In Dipoles  $b_{3,rms} = 8.1 \times 10^{-6} \text{ cm}^{-3}$   
 & F, QD  $b_{3,rms} = 1.5 \times 10^{-5} \text{ cm}^{-3}$

## $\gamma$ -shifts

(Reference AD/RHIC-22, H. Hahn)

For each Magnet type

$$\Delta\gamma_x = \frac{3}{4} \frac{NL}{\pi\rho} b_3 \left\{ \beta_x (x_{pf})^2 + \frac{1}{4} \beta_x^2 \epsilon_x - \frac{1}{2} \beta_x \beta_y \epsilon_y \right\}$$

$$\Delta\gamma_y = \frac{3}{4} \frac{NL}{\pi\rho} b_3 \left\{ -\beta_x (x_{pf})^2 - \frac{1}{2} \beta_x \beta_y \epsilon_x + \frac{1}{4} \beta_y^2 \epsilon_y \right\}$$

(4)

### $b_3$ effects

Largest  $\Delta V$  at  $\delta = 30^\circ$ ;  $E_{x_0} = 10$

$$\sigma_x \text{ (mm)} \quad 3.1$$

$$E_{x,N} \quad 3.3$$

$$E_x \quad 1.1$$

$$E_{b,\tau} = 6 E_x \quad 6.6$$

$$\sigma_p / 10^{-3} \quad 2.27$$

$$\frac{\sigma_0}{\rho} / 10^{-3} = \frac{2.500}{10^{-3}} \quad 5.7$$

$$\Delta V / 10^{-3} \quad 14. \quad 8.5 \text{ from dipoles}$$

To be Corrected

$$\Delta V / 10^{-3} \quad \rightarrow \quad \Delta V \text{ dipole reduced to } 4$$

$$b_3 L \text{ (m}^{-2}\text{)} \quad 30 \quad 16.5 \text{ for dipoles only}$$

Correction needed

$$\Delta V_{\text{spread}} = \Delta V \text{ for octupole, } \Delta V \sim \sigma p / \rho \text{ to even power.}$$

$$\text{Yellow book} \quad b_3 L = 170/\text{m}^2 \text{ at } B = 35 \text{ KG.}$$

$$\text{For } \epsilon_{2.58} = 1.1, \quad \sigma p / \rho = 2 \sigma_p = 4.54 \times 10^{-3}$$

$$\Delta V_{\text{spread}} = 5 \times 10^{-3} \text{ uncorrected } (\Delta V = 2.3 \text{ for dipoles})$$

$$\Delta V_{\text{spread}} = 2 \times 10^{-3} \text{ corrected}$$

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## 6σ question

Show we compute  $\Delta Y_x, \Delta Y_y$  for  $E_{6\sigma} = 6 E_{2.5\sigma}$ ?

No obvious need for 6σ requirement.

Requiring 6σ provides a safety factor to allow for unknown effects. This seems worthwhile.

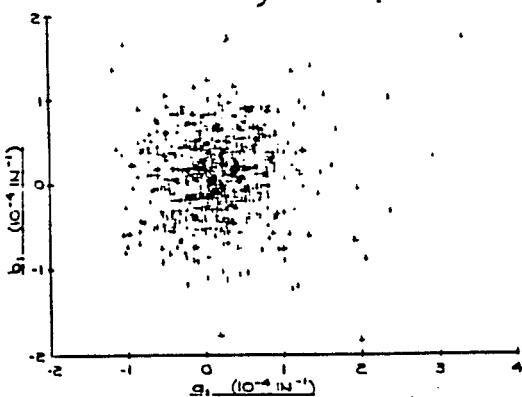
If we reduce the 6σ requirement  
~~(we do)~~ for  $\Delta Y$  due to  $b_3, b_4$ ,  
we reduce the safety factor.

## The 1/3 rule for ~~the~~ systematic $b_3$

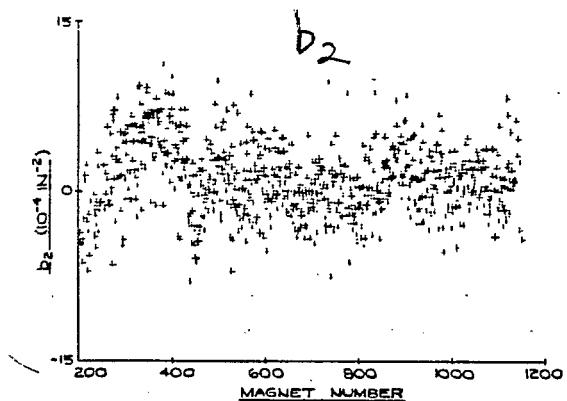
The next slide shows the Fermilab results that indicate

$$b_{3,av} = \frac{1}{3} b_{3,rms}$$

Can we make  $b_{3,av}$  smaller by  
watching  $b_3$  during the magnet  
construction?

Fig. 2.  $a_1$  and  $b_1$  coefficients

$72^\circ$  for the inner coil and  $36^\circ$  for the outer coil. The magnitude of  $b_4$  is also sensitive to these key angles. During collared coil fabrication we occasionally adjusted the key shims between the collars and the coils in order to maintain values of  $b_2$  within the acceptance range of  $0 \pm 6$  units at 4000 A excitation. In Figure 3 we show  $b_2$  at 4000 A as a function of magnet number, which closely approximates the date of collared coil construction. Note that a trend toward unacceptably large values of  $b_2$  developed

Fig. 3.  $b_2$  coefficient at 4000 A as a function of magnet number (construction date)

during startup of the production line; the underlying construction problems were identified and brought under control after magnet number 410; and as the figure shows, thereafter we were able to maintain the average value of  $b_2$  near zero.

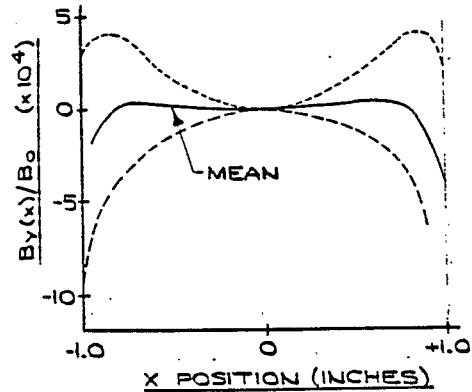
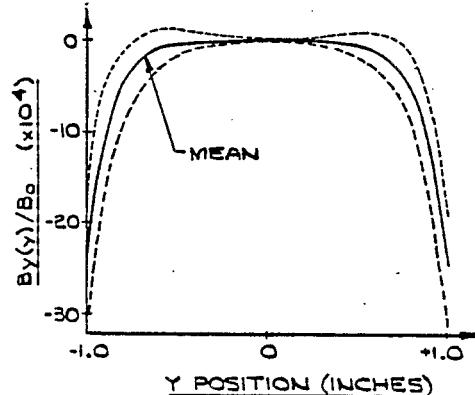
| Pole Index<br>$n$ | $b_n$<br>Design | $b_n$<br>Mean | $b_n$<br>RMS | $a_n$<br>Mean | $a_n$<br>RMS |
|-------------------|-----------------|---------------|--------------|---------------|--------------|
| 1                 |                 | 0.09          | 0.48         | 0.17          | 0.50         |
| 2                 | .04             | 0.95          | 3.12         | 0.10          | 1.16         |
| 3                 |                 | -0.23         | 0.77         | -0.07         | 1.46         |
| 4                 | 1.04            | -0.57         | 1.32         | -0.10         | 0.46         |
| 5                 |                 | -0.07         | 0.32         | -0.07         | 0.55         |
| 6                 | 4.44            | 5.48          | 0.54         | -0.07         | 0.29         |
| 7                 |                 | 0.04          | 0.17         | 0.22          | 0.26         |
| 8                 | -12.09          | -12.52        | 0.33         | -0.07         | 0.41         |
| 9                 |                 | 0.02          | 0.23         | 0.28          | 0.38         |
| 10                | 3.63            | 3.70          | 0.26         | 0.08          | 0.25         |
| 11                |                 | -0.01         | 0.20         | -0.24         | 0.25         |
| 12                | -0.82           | -0.80         | 0.19         | -0.05         | 0.22         |

Table 1 Harmonic coefficients of the magnetic field at 4000 A in standard units for 870 dipoles

In Table 1 we show a table of values of harmonic coefficients at 4000 A excitation up to the 26-pole together with the original design estimates for  $b_n$ ,  $n$  even, made for 4500 A. The  $n=7$  data are artifacts of the data handling procedure.<sup>3</sup> The rms values shown for higher order  $a_n$  and  $b_n$ ,  $n$  odd, probably indicate the measurement precision. The widths of the distributions of  $b_2$ ,  $a_2$ , and  $a_3$ , which represent real magnet to magnet variation, are relatively large; as a consequence dipoles are assigned locations in the accelerator to reduce undesirable orbit effects that may arise from these field components.<sup>4</sup>

#### FIELD SHAPE

Another way to characterize the magnetic field is to show  $B_y/B_0$  along the x and y axes out to  $\pm 1$  inch. Figures 4 and 5 show these distributions. The mean value of  $B_y/B_0$  for the sample of 870 dipoles at each x or y is traced by the solid lines, and the dotted lines give a band containing 90% of the magnets.

Fig. 4.  $B_y/B_0$  as a function of x at  $y=0$ Fig. 5.  $B_y/B_0$  as a function of y at  $x=0$ 

#### INTEGRAL FIELD

Integral field measurements were made using stretched wire loops at many magnet excitation currents. Figure 6 shows the distribution of normalized field integral at 2000 A magnet excitation as a function of magnet number.

It can be seen that the construction problems that affected  $b_2$  also affected the integral field. These same data are shown as a histogram in Figure 7;

IEEE, 1983 - Fermilab Dipoles

## The Systematic $a_1$ effect

The systematic  $a_1$  implied by the  $1/3$  rule gives a large  $\Delta\nu$  splitting.

$a_{1,\text{av}} = \frac{1}{3} a_{1,\text{rms}}$  in the dipoles gives a  $\Delta\nu$  splitting due to  $a_{1,\text{av}}$  in the dipoles which is 2 times the  $\Delta\nu$  splitting due to  $a_{1,\text{rms}}$  in the dipoles.

Total  $\Delta\nu$  splitting, due to both dipoles and quads, is increased by factor 2.4 for  $B^* = 6$  by the  $a_{1,\text{av}} = (1/3) a_{1,\text{rms}}$ .

This will increase the  $a_1$  correction requirement by about a factor of 2.

This probably will increase the number of correction skew quads.

Another problem is the residual  $\Delta\nu$  splitting after correction due to higher order in  $a_1$  effects. At present it is  $\Delta\nu$  splitting  $= 15 \times 10^{-3}$  in the worst case. This may be increased by factor 2 by  $a_{1,\text{av}} = (1/3) a_{1,\text{rms}}$ . However, it is possible that  $a_{1,\text{av}}$  does not affect the residual  $\nu$ -splitting.

Recommendation: Some way be found to keep  $a_{1,\text{av}}$  small;  $a_{1,\text{av}} \leq .1$   $\text{rms}$  at least.

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A possible solution of the problem of  $a_{1,av}$  in the dipoles, suggested by H. Hahn, is to use the skew quads in the arcs to generate an  $a_{1,av}$ . This would require 4 families of skew quads in each sextant instead of the presently planned 2 families.

|                 |   |   |   |    |    |   |   |    |    |   |   |    |    |
|-----------------|---|---|---|----|----|---|---|----|----|---|---|----|----|
| old<br>2-family | → | 1 | 2 | -1 | -2 | 1 | 2 | -1 | -2 | 1 | 2 | -1 | -2 |
|                 | — | ① | ① | ①  | ①  | ① | ① | ①  | ①  | ① | ① | ①  | —  |
| New<br>4 family | — | 1 | 2 | 3  | 4  | 1 | 2 | 3  | 4  | 1 | 2 | 3  | 4  |

① indicate QD quads

This can reduce the  $a_{1,av}$  in dipoles effect by about factor 5. (The imaginary part of the coupling stopband, about 20% of the Real part, cannot be canceled).

The extra families may also be useful for reducing two effects of the random  $a_1$ , the residual V-splitting and the reduction in dynamic aperture.